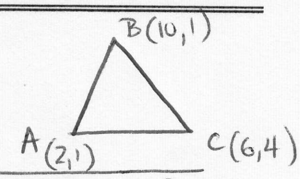
Exam Practice Questions – Day 2



1) A triangle has vertices A(2, 1), B(10, 1) and C(6, 4).

a) **Verify** that the triangle is isosceles using a method of your choice. **Show your work.**

b) **Describe** other methods you could have used to answer question a).

$d_{AC} = \sqrt{(6-2)^2+(4-1)^2}$
$= \sqrt{(4)^2+(3)^2}$
$= \sqrt{16+9}$
$= \sqrt{25}$
$= 5$

$d_{AB} = \sqrt{(10-2)^2+(1-1)^2}$
$= \sqrt{(8)^2+(0)^2}$
$= \sqrt{64}$
$= 8$

$d_{BC} = \sqrt{(10-6)^2+(1-4)^2}$
$= \sqrt{(4)^2+(3)^2}$
$= \sqrt{16+9}$
$= \sqrt{25}$
$= 5$

Since two sides are the same length it is a isosceles.

2) The sine and cosine values are the same for some pairs of angles. For example, for the pair of angles 20° and 70°, the sine of 20° is the same as the cosine of 70° ($\sin 20° = \cos 70°$).

a) **State** two other pairs of angles for which this works.

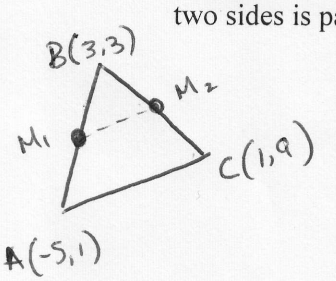
b) What is the pattern?

a) $\sin 30° = 0.5$ & $\sin 15° = 0.2588$
$\cos 60° = 0.5$ $\cos 75° = 0.2588$ } two other pairs

b) The pattern is that both angles add up to 90°.

$\boxed{\sin A = \cos(90° - A)}$

3. a) Triangle ABC has vertices A(-5, 1), B(3, 3) and C(1, 9). Show that the line segment joining the midpoints of two sides is parallel to the third side.



$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{-5+3}{2}, \frac{1+3}{2}\right)$
$= \left(\frac{-2}{2}, \frac{4}{2}\right)$
$= (-1, 2)$

$M_{BC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{3+1}{2}, \frac{3+9}{2}\right)$
$= \left(\frac{4}{2}, \frac{12}{2}\right)$
$= (2, 6)$

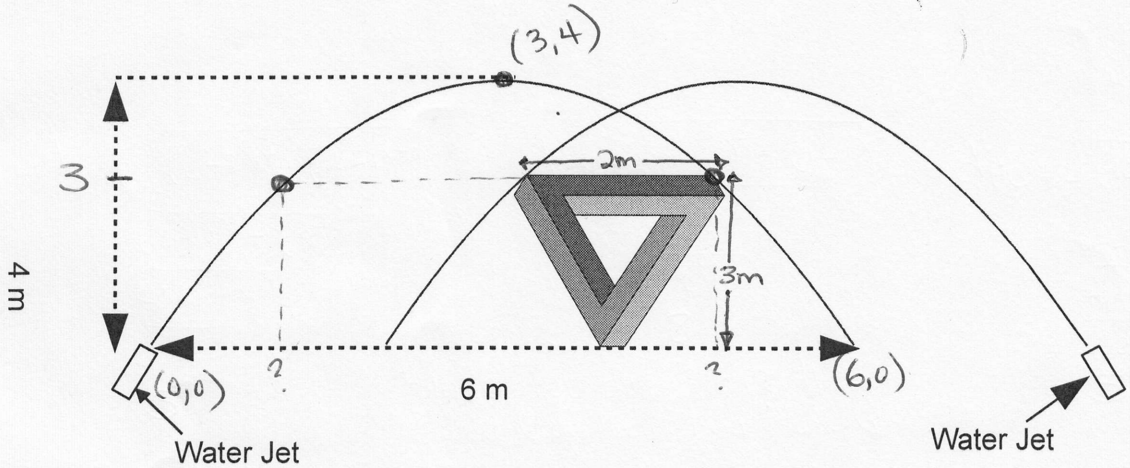
$\text{Slope}_{M_1M_2} = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{6-(2)}{2-(-1)}$
$= \frac{4}{3}$

$\text{Slope}_{AC} = \frac{y_2-y_1}{x_2-x_1}$
$= \frac{9-(1)}{1-(-5)}$
$= \frac{8}{6}$ ÷2 Reduce
$= \frac{4}{3}$

b) Do you think the property from part a) is true for *all* triangles?

Yes by using the same method as above.

4) A beautiful water fountain is created by projecting identical parabolic shaped streams of water into the air, as shown below. Each stream of water reaches a height of 4 m, and has a horizontal width of 6 m.



A triangular statue, that is 2m wide and 3m high, is placed between two water jets as shown. The water jets need to be positioned so that the statue will remain dry. **Determine** the possible positions of the water jets. **Justify your answer**. (Diagram is not drawn to scale)

$y = a(x-3)^2 + 4$

Sub (0,0) to find a

$0 = a(0-3)^2 + 4$

$-4 = a(-3)^2$

$\frac{-4}{9} = \frac{9a}{9}$

$a = -\frac{4}{9}$

Vertex equation

$y = -\frac{4}{9}(x-3)^2 + 4$

☆ interested in height of statue $y = 3$

sub $y = 3$ into vertex form

$y = -\frac{4}{9}(x-3)^2 + 4$

$3 = -\frac{4}{9}(x-3)^2 + 4$

$\boxed{\frac{3-4}{-4/9}} = \frac{-4/9(x-3)^2}{-4/9}$

$\boxed{\begin{aligned} &= -1 \div -4/9 \\ &= -1 \times -\tfrac{9}{4} \\ &= +\tfrac{9}{4} \end{aligned}}$

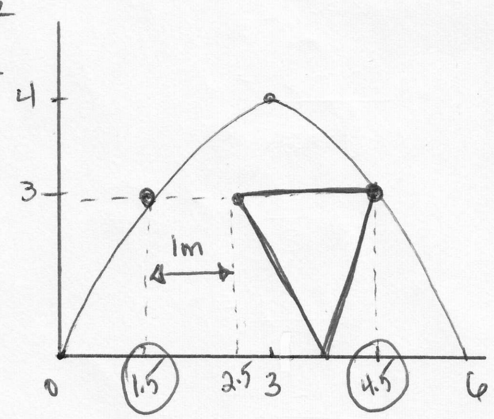
$\sqrt{\frac{9}{4}} = \sqrt{(x-3)^2}$

$\pm 1.5 = x - 3$

$3 \pm 1.5 = x$

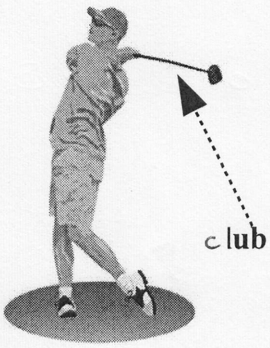
$x = 3 + 1.5$ $\quad$ $x = 3 - 1.5$

$x = 4.5$ $\quad$ $x = 1.5$

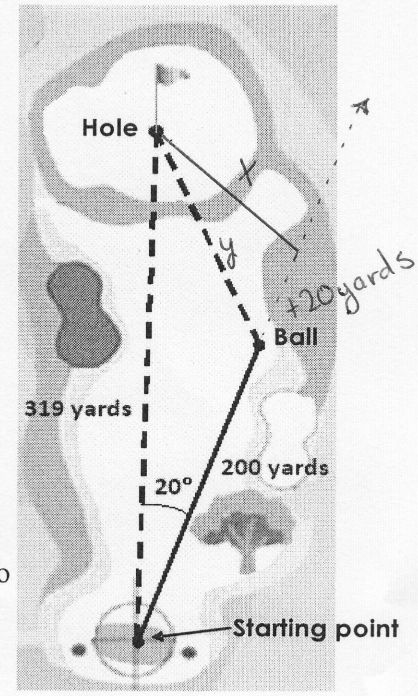


∴ the water jets can move 1m closer to the statue on both sides.

5. In golf, clubs are designed to hit the ball different distances. The table below summarizes the club distances for Arnold.



Club Name	Distance Range (yards)
2 iron club	190 - 220
3 iron club	180 - 200
5 iron club	165 - 180
7 iron club	145 - 170
8 iron club	130 - 150
9 iron club	110 - 135



Arnold used a 2 iron for his first shot. The ball travelled 200 yards from the starting point but it was 20° off line. Which club should Arnold choose for his second shot so that it gets as close to the hole as possible? **Justify** your answer.

$y^2 = 319^2 + 200^2 - 2(319)(200)\cos 20°$

$\sqrt{y^2} = \sqrt{21856.22159}$

$y \doteq 147.84$

$x^2 = 319^2 + 220^2 - 2(319)(220)\cos 20°$

$\sqrt{x^2} = \sqrt{18265.74375}$

$x \doteq 135.15$

If arnold uses the same club but hits it 220 yards (harder) he will get 12.69 yards closer to the hole.

3. A teacher asks his class to come up with a model for a quadratic relation that has a maximum of 8 and zeros of −2 and 6. Below are three students' models.

Ahmed's Model

1st

x	y	
-3	-2	
-2	0	0-(-2)=2
-1	2	2-(0)=2
0	4	4-2=2
1	6	6-4=2
2	8	8-6=2
3	6	6-8=-2
4	4	4-6=-2
5	2	2-4=-2
6	0	0-2=-2
7	-2	-2-0=-2

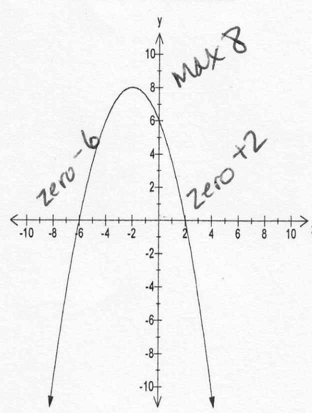
Zeros (x = -2)

max 8 Vertex (x = 2)

zeros (x = 6)

Not Quadratic

Bo's Model



Cindy's Model

$y = \frac{1}{2}(x-2)^2 - 8$

a = + opens up
min -8

$= \frac{1}{2}(x-2)(x-2) - 8$

$= \frac{1}{2}(x^2 - 4x + 4) - 8$

$= \frac{1}{2}x^2 - 2x + 2 - 8$

$= \frac{1}{2}x^2 - 2x - 6$

$= \frac{1}{2}(x^2 - 4x - 12)$

Give each student feedback by **comparing** their model to the quadratic relation described by the teacher.

$= \frac{1}{2}(x-6)(x+2)$

Ahmed
- zeros are correct
- Vertex k value is 8
- Not Quadratic
  - 2nd differences will not be constant

Bo's
- zeros are wrong
- opens down
- max of 8 (k value)

Cindy
- zeros correct
- open up
- min of 8 (k value)
- Vertex (2,-8)

$x-6=0$ OR $x+2=0$

$\boxed{x=6}$ $\boxed{x=-2}$